Coppercloud: Blind Server-Supported RSA Signatures

26.04.26

Nikita Snetkov Jelizaveta Vakarjuk Alisa Pankova

2002: Identity cards

- Create signatures, protecting the private keys
- User's computer must have a card reader
- There are more-or-less standard software components for browsers and other applications to access the card
- To activate the card, it must receive a PIN from the software component or the card reader
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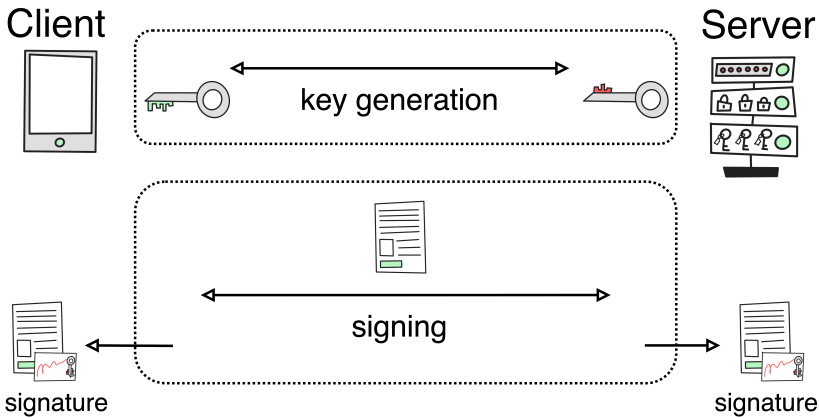
Would it be a good idea to store the private key in a device like smartphone?

2016: SplitKey: the main idea



[1]image author: Jelizaveta Vakarjuk

RSA public key encryption

$$p, q \leftarrow\$ \mathbb{P}$$
$$n \leftarrow p \cdot q$$
Choose an integer e such that $1 < e < \phi(n)$ and $gcd(e, \phi(n)) = 1$ //public key
$$d \leftarrow e^{-1} \pmod{\phi(n)} \quad \text{//private key}$$

where $\phi(n) = (p-1)(q-1)$ is the Euler's totient function

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Server

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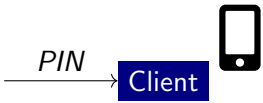

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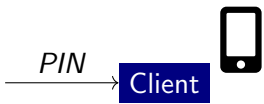

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$\mathsf{pk} = (n_1 n_2, e)$

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$u,\ n_1,\ n_2$

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$d_1'',\ d_2,\ n_1,\ n_2$

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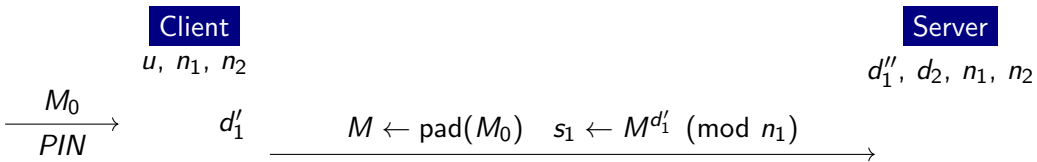

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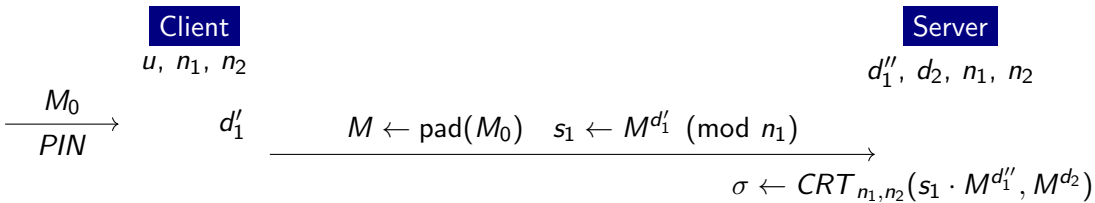

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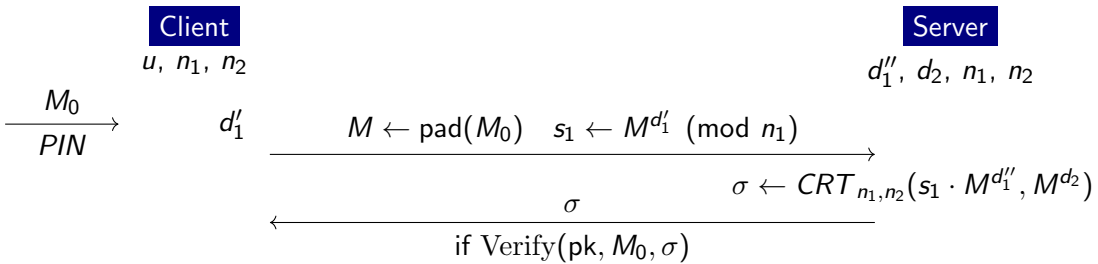

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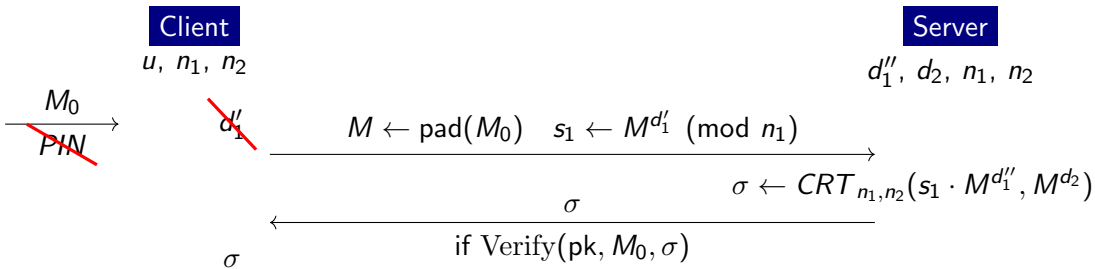

Clone detection

- Additionally, Client and Server share a random bitstring w
- During signing, Client also sends w to Server
- Server checks that the value of w is as expected
  - If not, Server disqualifies this Client / this pk
- Server generates a new w and sends it back, together with σ

SplitKey signing (without blinding)

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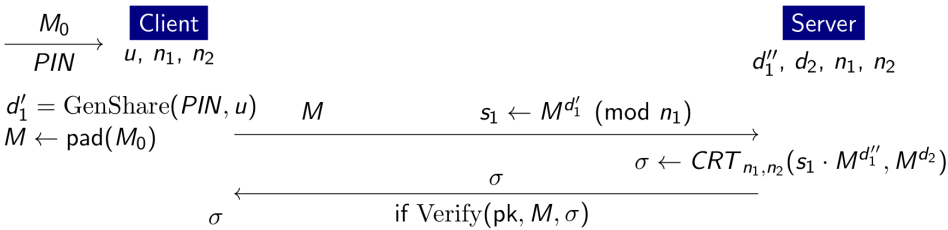

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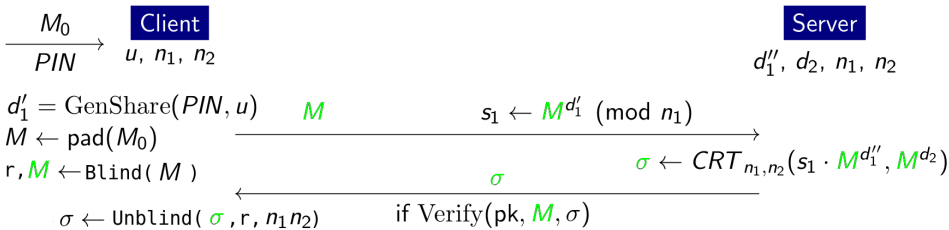
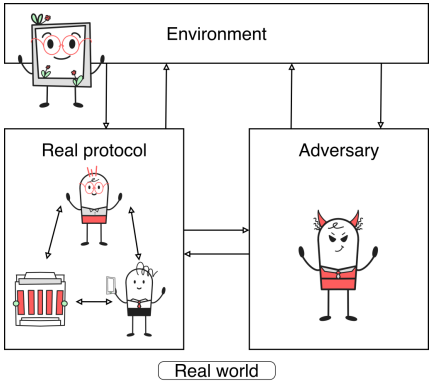


Proofs in the Real-Ideal world paradigm (e.g. Universal Composability)



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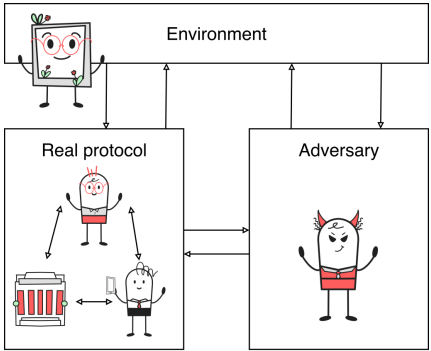

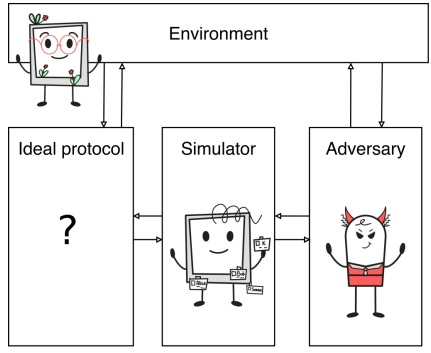

How should one define the ideal functionality?

[2]image author: Jelizaveta Vakarjuk

Interactions that need to be simulated (for signing)

- Signing by the Client (honest) and the Server (honest)
- Signing by the Client (honest) and the Adversary – corrupted server case
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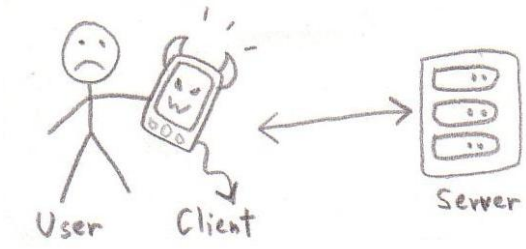
  - What a corrupted client should be able to do?



Possible client corruptions that we want to model for SplitKey

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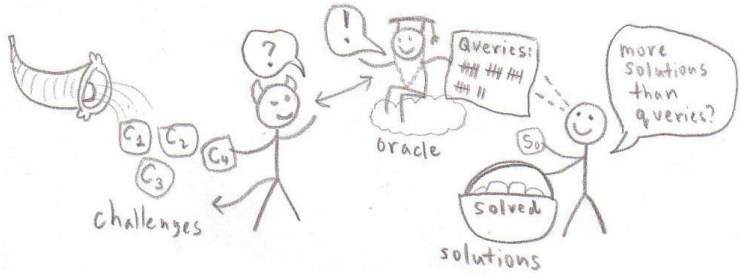
Proving a limit on the number of signatures a client can generate

One-more cryptographic assumptions:

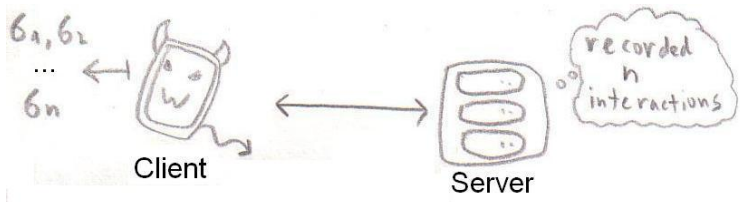
- The adversary is given many instances of a cryptographic challenge (e.g. to solve the RSA problem).
- The adversary can query an oracle $O(\cdot)$ that allows to solve the RSA probem (on any input of adversary's choice).
- The adversary wins if it solves $n+1$ different challenges while quering $O(\cdot)$ at most n times.



Proving a limit on the number of signatures a client can generate

RSA-ACTI (Alternative Chosen-Target RSA Inversion) problem: [Bellare, 2002].

- Intuitively, getting n different RSA signatures from the server does not allow the adversary to somehow obtain one more signature.
- The adversary gets as many signatures as there have been signing sessions.
- Under RSA-ACTI assuptions, the client should not be able to sign more messages than approved by the server.



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- The (un)blinding can be performed locally by the client, without the need to modify server's computation, or the signature verification.
- The ideal functionality needed more modifications than the real protocol.
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  - Non-blinded signing: a signature σ on m is generated only if approved by the server.
  - Blinded signing: the client should not be able to sign more messages than approved by the server.
- ...the non-blinded version does not guarantee that the adversary cannot somehow magically obtain one more signature.
- In practice, the adversary would need one more sensible signature, which can be much harder.

Thank You!