Special Soundness and Binding Properties: A Framework for Tightly Secure zk-SNARKs

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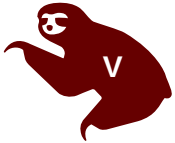
UNIVERSITY OF TARTU

Main goal

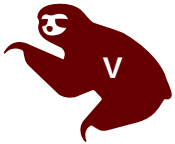
To provide a new proof technique for proving that a constructed SNARK is knowledge sound with a tight error bound.

Interactive Argument



Interactive Argument

Statement x, witness w
x holds iff $\exists w : (x, w) \in R$



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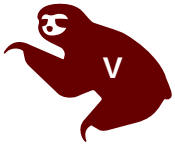

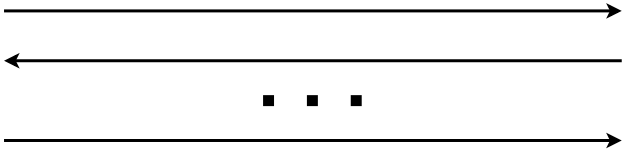

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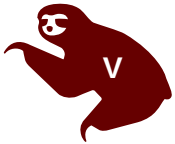

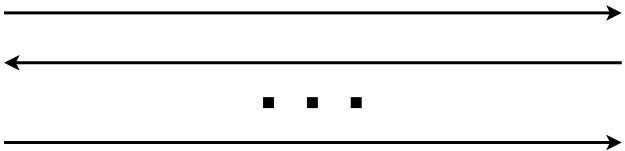

Accept or reject

Non-Interactive Argument

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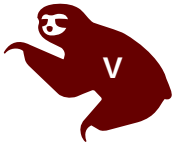

π

Accept or reject

Succinct Non-Interactive Argument of Knowledge (SNARK)

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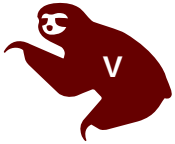

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- **Succinct** – π is small compared to w; verification is fast.

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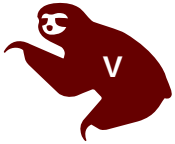

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$$\text{SNARK} = \text{PIOP} + \text{PCS} + \text{Fiat Shamir transformation.}$$

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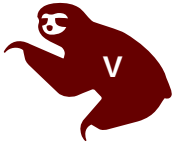

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Polynomial Interactive Oracle Proof (PIOP)

Challenge phase

Statement x, witness w

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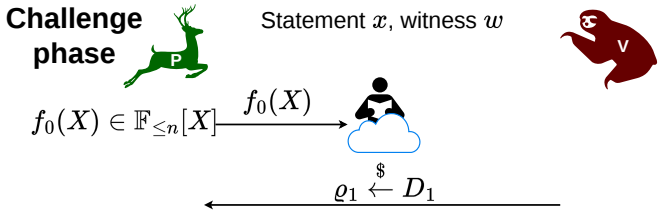
Challenge phase

Statement x, witness w

$f_0(X) \in \mathbb{F}_{\leq n}[X]$ $\xrightarrow{f_0(X)}$

Polynomial Interactive Oracle Proof (PIOP)



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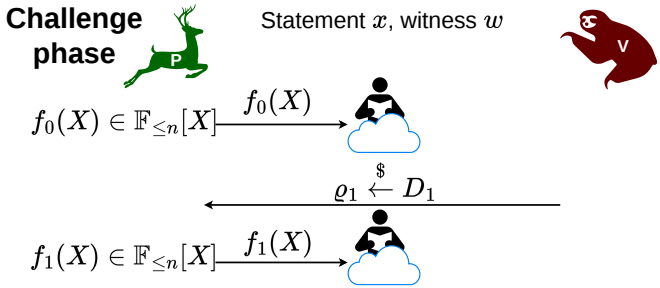

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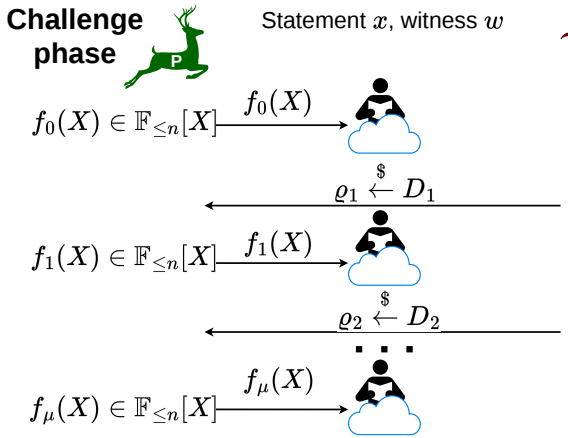

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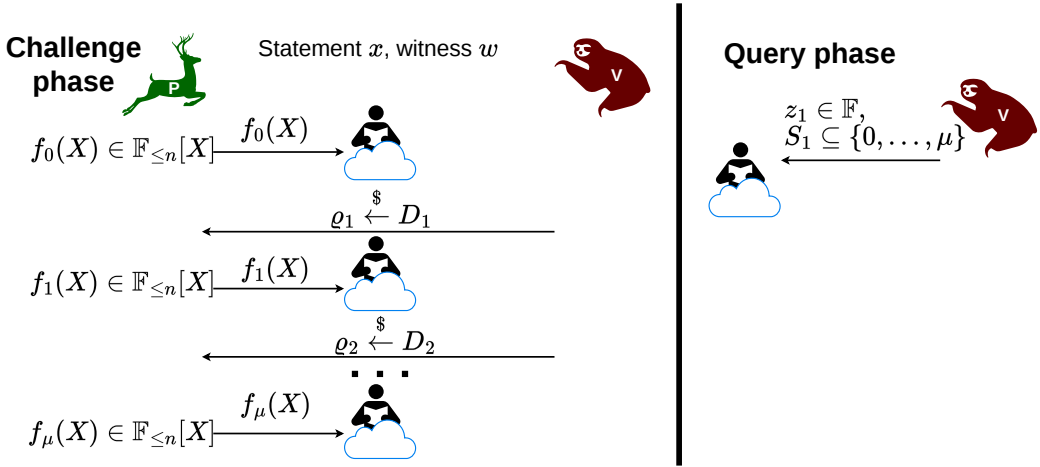

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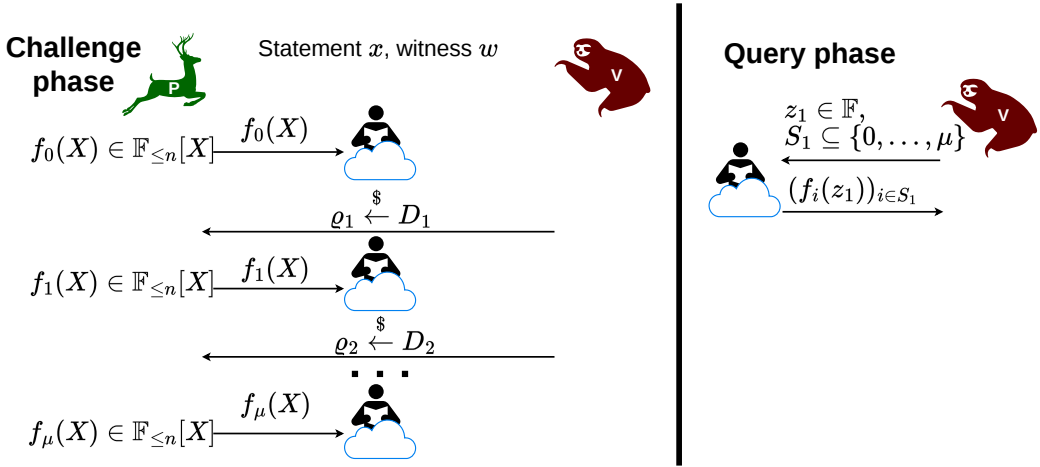

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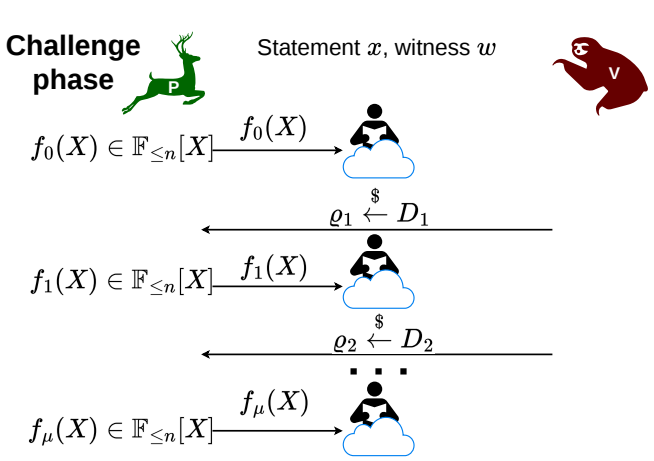

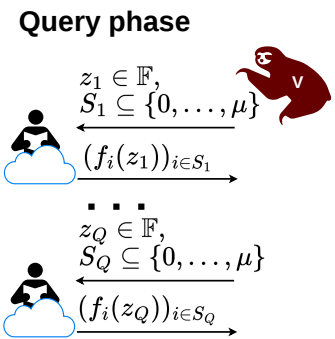

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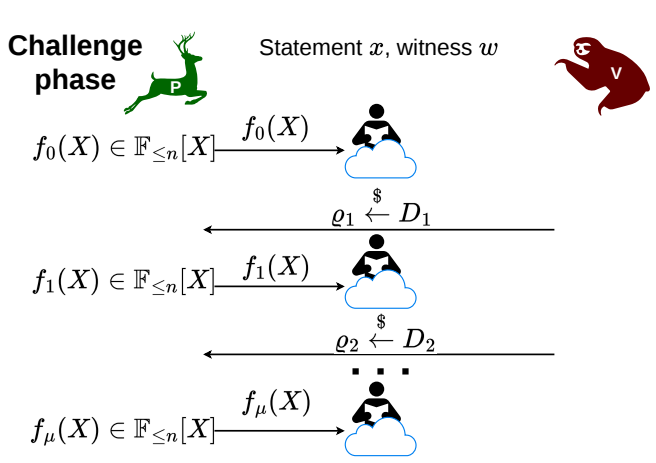
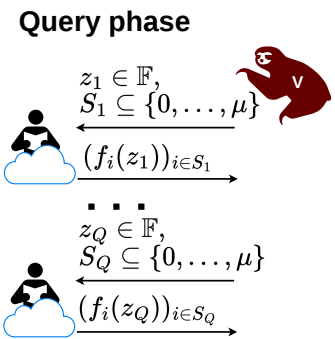




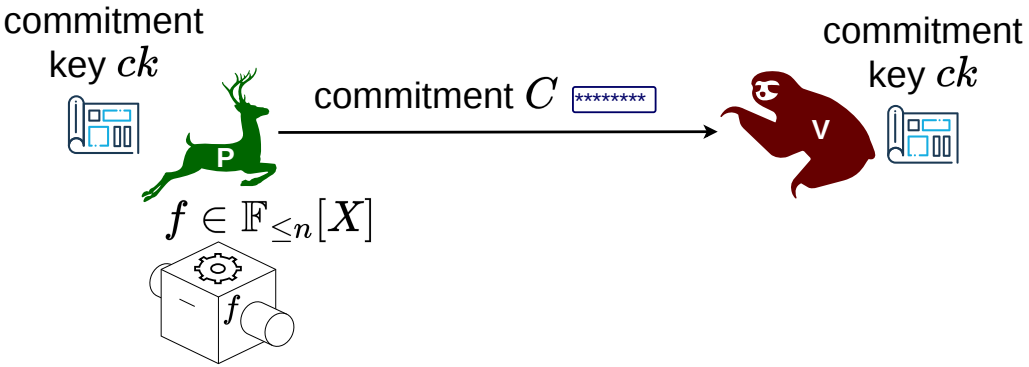
Requirements for the Oracle

We require from the oracle  to

1. accept polynomials with degree $\leq n$ in the challenge phase;
2. input a field element and a set of indices, and return the evaluations of the polynomials specified by the indices in the query phase.

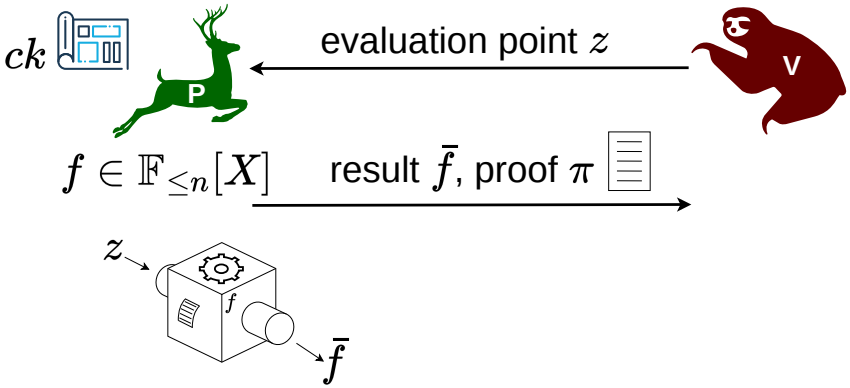
Polynomial Commitment Scheme (PCS)

Commitment



Polynomial Commitment Scheme (PCS)

Opening



Polynomial Commitment Scheme (PCS)

Verification

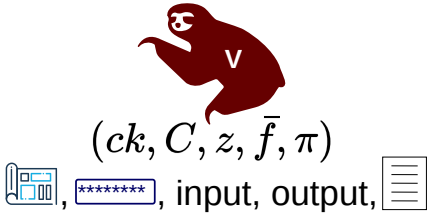
$(ck, C, z, \bar{f}, \pi)$


, ********, input, output,


Accept
or reject

Polynomial Commitment Scheme (PCS)

Verification



Note that there exist such commitment schemes. For example, see [KZG10].

Soundness Notions for Interactive Arguments

- **Knowledge soundness** – There exists an extractor such that if a prover P can convince the verifier V with high enough probability that the statement holds, then with high probability the extractor can extract a witness w.

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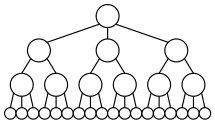
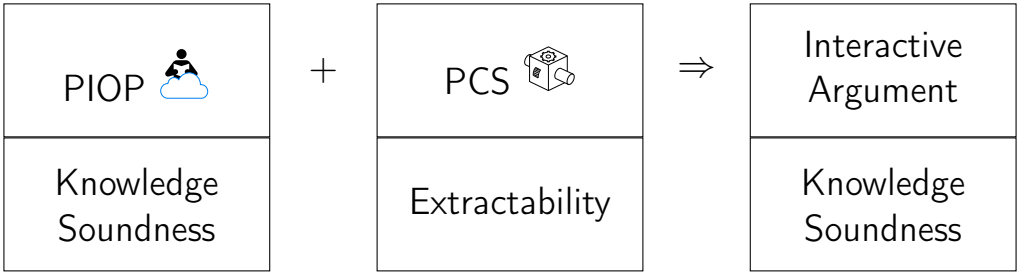
- **Special soundness** – There exists an extractor such that, given a tree of accepting transcripts, it can output a witness w.

Tree of accepting transcripts → Extractor → witness w



Motivation

Previous work



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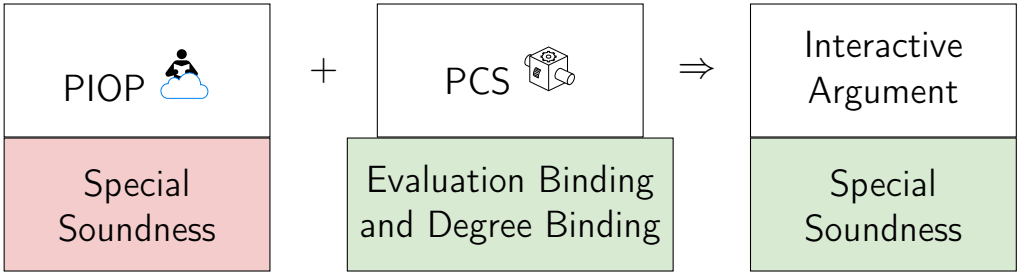
Our work



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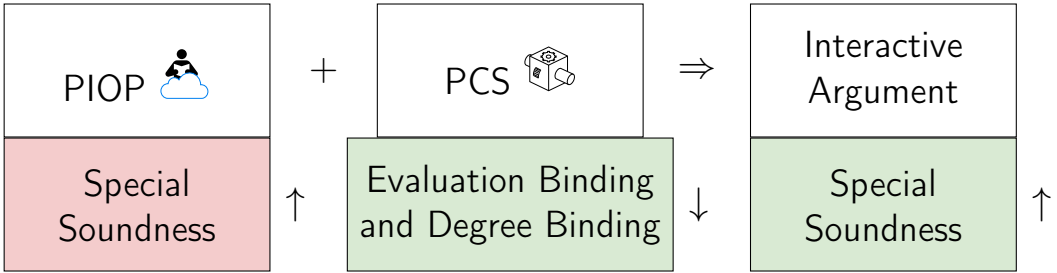

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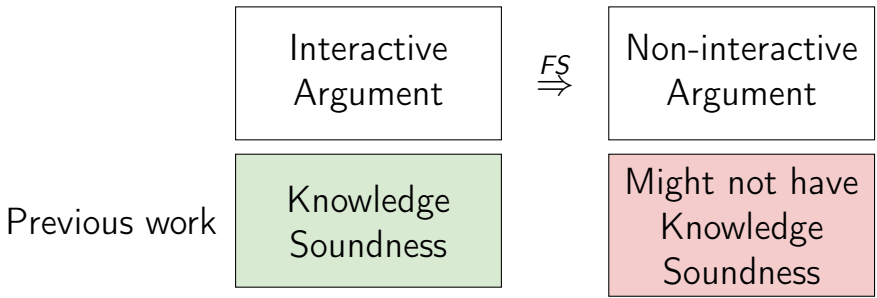
Interactive Argument
FS
Non-interactive Argument
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Knowledge Soundness
Might not have Knowledge Soundness

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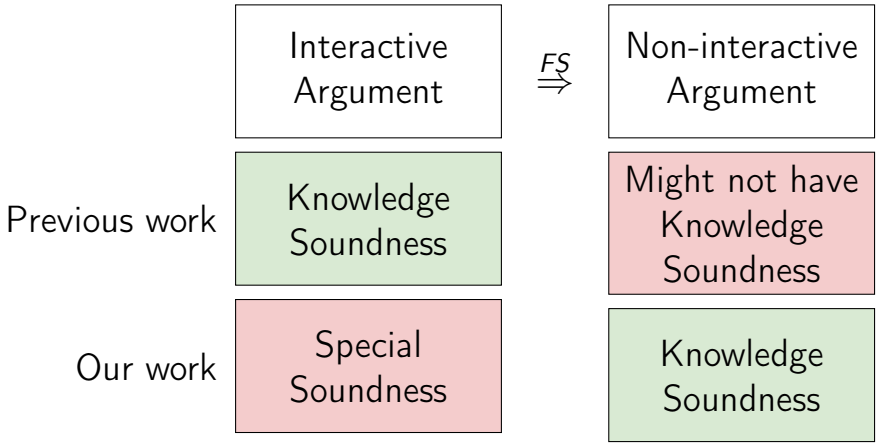
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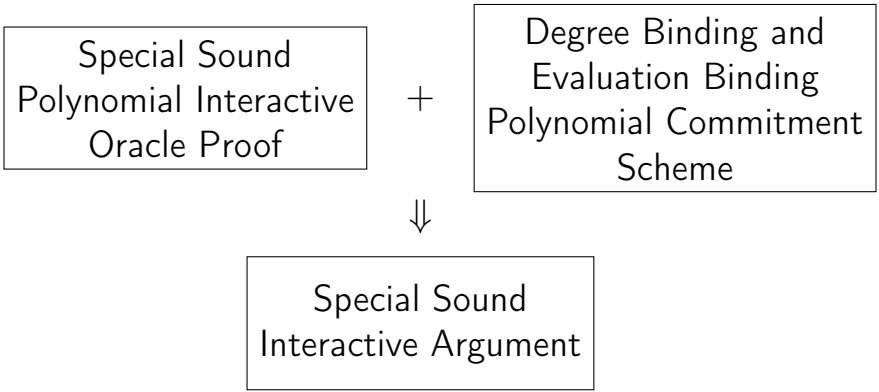
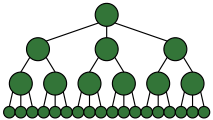
Knowledge Soundness

Main Theorem



Special Soundness for Polynomial Interactive Oracle Proof

Tree of accepting transcripts



→

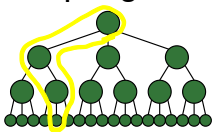
Extractor

→

Witness w

Special Sound Polynomial Interactive Oracle Proof

Tree of accepting transcripts



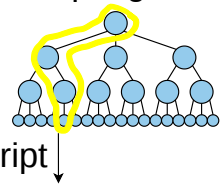
Extractor

Witness w

$$\text{transcript}_{\text{PIOP}} = \left(f_1, \varrho_1, \ldots, f_\mu, \varrho_\mu, f_{\mu+1}, \left(z_i, \bar{f}_{S_i}\right)_{i=1}^{Q}\right) \quad .$$

Special Soundness for Interactive Argument

Tree of accepting transcripts



Extractor

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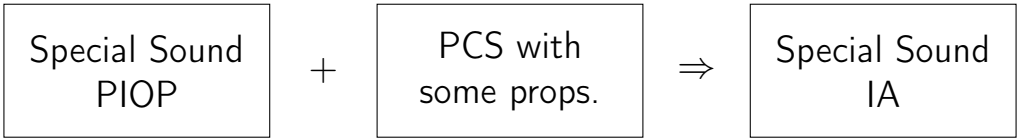
$$\text{transcript}_{\mathsf{IA}} = \left(C_1, \varrho_1, \ldots, C_\mu, \varrho_\mu, C_{\mu+1}, (z_i, \bar{f}_{S_i}, \pi_{S_i})_{i=1}^{Q}\right) \quad .$$

Special Sound Interactive Argument



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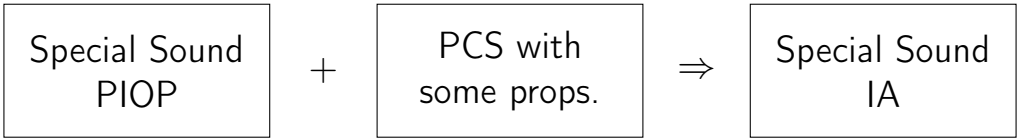
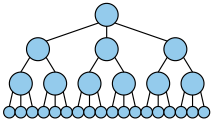


We need to construct Interactive Argument Extractor  . Note that we have access to the PIOP Extractor  .

Main Theorem Proof Sketch

Tree of accepting transcripts



➔

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➔

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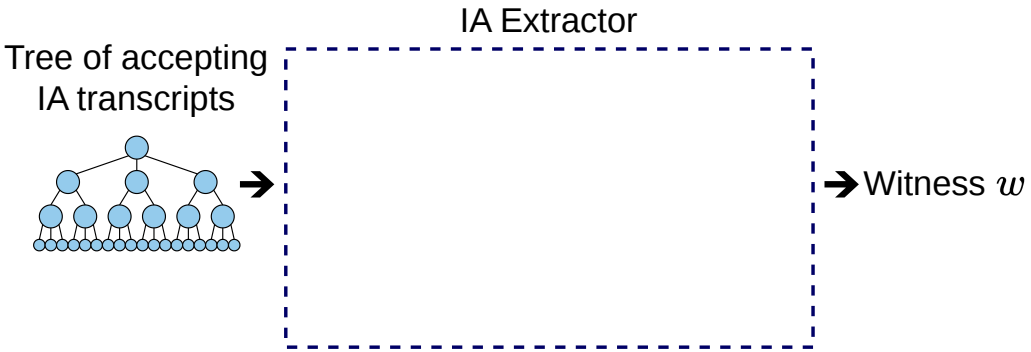

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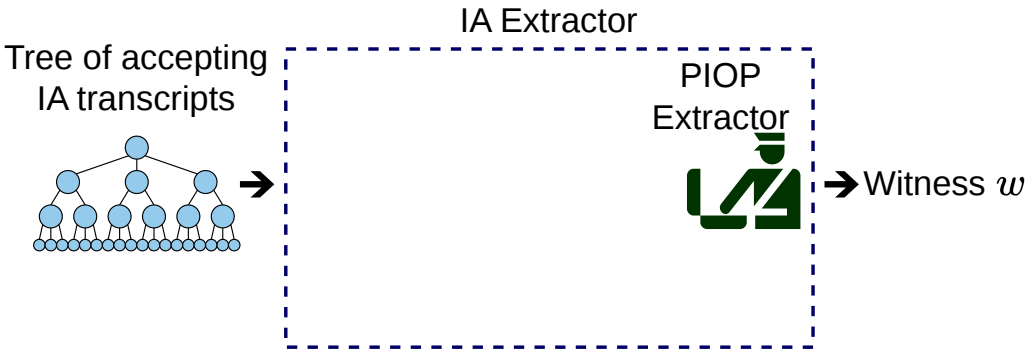

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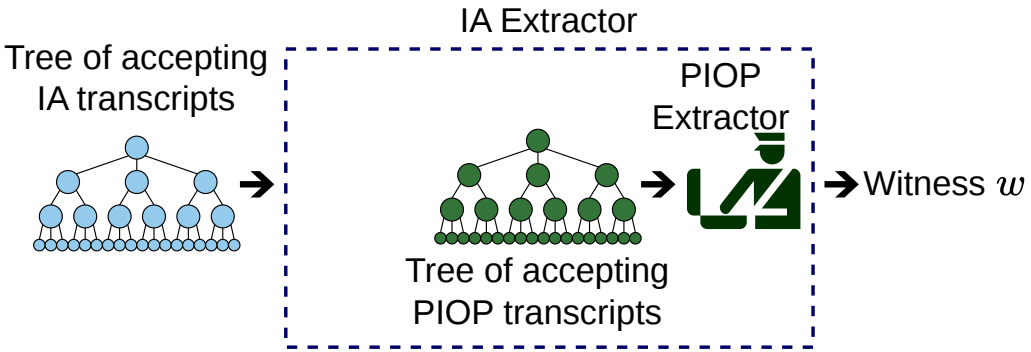

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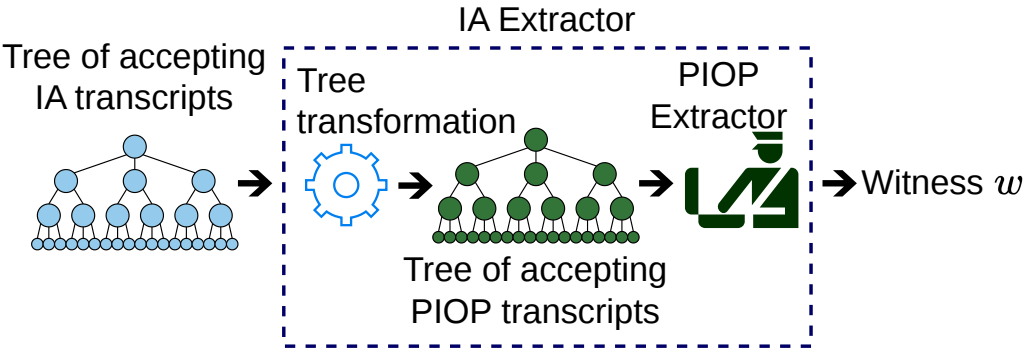
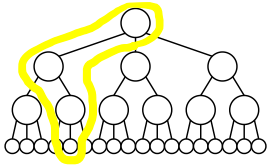


Constructing the Tree Transformation: Transcripts



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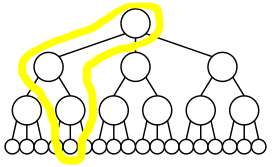

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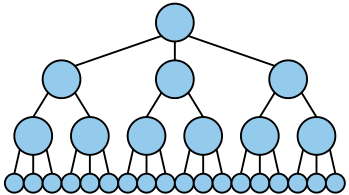
Recovering Polynomials from the Tree of Accepting Transcripts

Fix a commitment C and choose all places in the tree, where the corresponding polynomial is queried.



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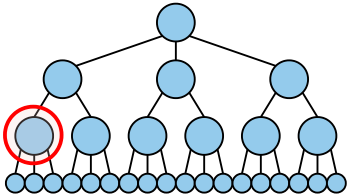

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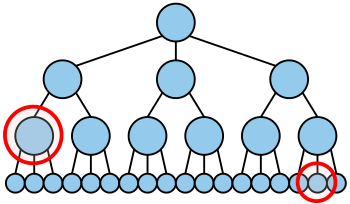

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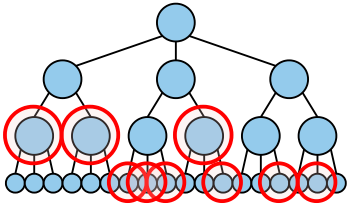

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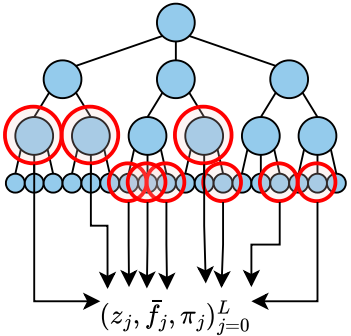

Recovering Polynomials from the Tree of Accepting Transcripts

We have $(z_j, \bar{f}_j, \pi_j)_{j=1}^{L}$.

- What if $z_i = z_j$ but $\bar{f}_i \neq \bar{f}_j$?

Recovering Polynomials from the Tree of Accepting Transcripts

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- What if $z_i = z_j$ but $\bar{f}_i \neq \bar{f}_j$?
- This means the underlying *polynomial* behind the commitment is not a function.

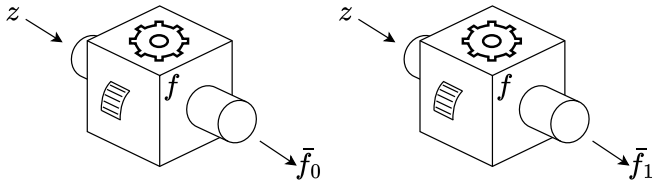
Recovering Polynomials from the Tree of Accepting Transcripts

We have $(z_j, \bar{f}_j, \pi_j)_{j=1}^{L}$.

- What if $z_i = z_j$ but $\bar{f}_i \neq \bar{f}_j$?
- This means the underlying *polynomial* behind the commitment is not a function.
- Assume it is hard to find such a situation.

Evaluation Binding

- It should be difficult for a probabilistic polynomial time adversary $\mathcal{A}$ to find $(C, z, \bar{f}_0, \pi_0, \bar{f}_1, \pi_1)$ such that V accepts and $\bar{f}_0 \neq \bar{f}_1$.



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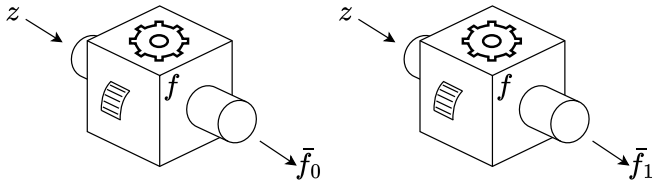

- Oracle holds functions that give one output per input.

Recovering Polynomials from the Tree of Accepting Transcripts

We have $(z_j, \bar{f}_j, \pi_j)_{j=1}^{L}$ and for all $z_i = z_j$ it holds $\bar{f}_i = \bar{f}_j$, thus we can interpolate and get $\tilde{f} \leftarrow \text{Interpolate}((z_j, \bar{f}_j)_{j=0}^{L})$.

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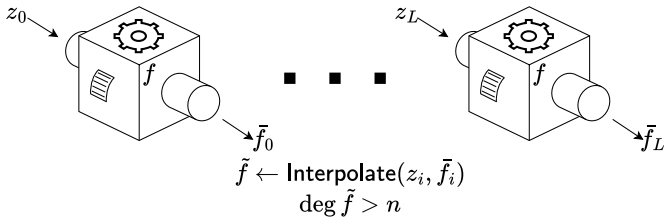

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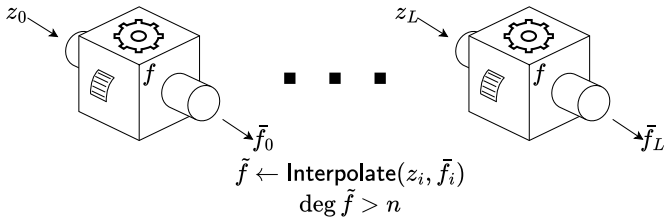

- Oracle holds polynomials with degree up to n.

Constructing the Tree Transformation

$$\text{transcript}_{\text{IA}} = \left(C_1, \varrho_1, \ldots, C_\mu, \varrho_\mu, C_{\mu+1}, (z_i, \bar{f}_{S_i}, \pi_{S_i})_{i=1}^{Q}\right) \quad ,$$

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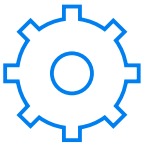

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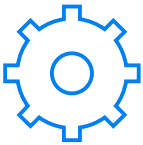

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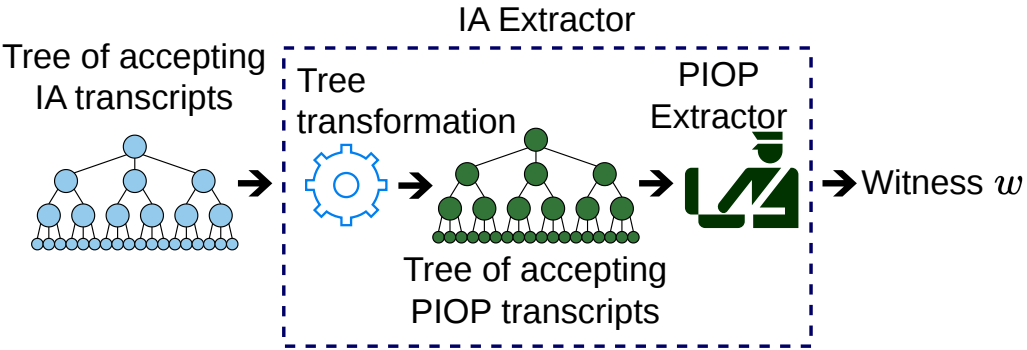
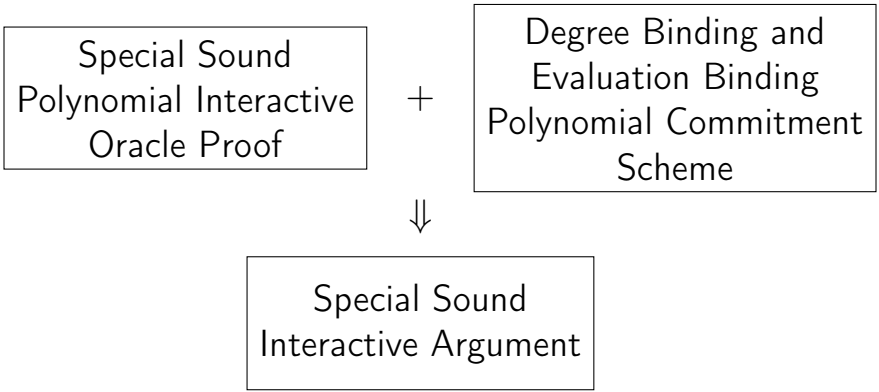
Main Theorem Proof Sketch



Main Theorem



Other Results from the Paper

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- Binding properties of KZG Polynomial Commitment Scheme and its optimizations.

See [KLPS26] for the details.

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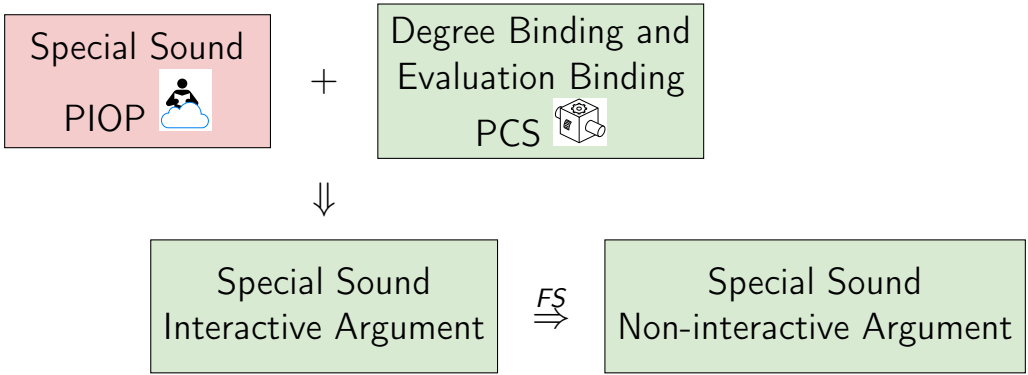
  - Can we make a similar theory for stronger security notions like simulation extractability and universal composability?

Thank you for listening!

Questions


Special Sound
PIOP
+
Degree Binding and
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PCS
⇓
Special Sound
Interactive Argument
FS
⇒
Special Sound
Non-interactive Argument

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